UNIVERSITATEA "AL. I. CUZA" IASI
FACULTATEA DE MATEMATICA

LUCRARILE CONFERINTEI NATIONALE DE GEOMETRIE SI TOPOLOGIE
Suceava, 6-9 octombrie, 1988
Iasi, 1989

The Model of an Elementary Boolean Geometry. Boolean Complex Numbers

Serban E. Vlad
Str. Zimbrului, Nr.3, Bl.PB68, Et.2, Ap.11, 3700, Oradea, Romania
serban_e_vlad@yahoo.com

We give the model for an unknown theory, which is left as an open problem.

1) $\boldsymbol{B}_2 = \{0,1,\oplus,\cdot\}$ is the binary Boole algebra, where '$\oplus$' is the modulo 2 sum and '$\cdot$' is the intersection. For $n \in \{2,3\}$, $\boldsymbol{B}_2^n$ is the *space*, $x, y,... \in \boldsymbol{B}_2^n$ are the *points*, $\{x, y\}, x \neq y$ are the *lines*. The addition (x, y) **a** $\overline{xy} \overset{def}{=} x \oplus y$ makes $\boldsymbol{B}_2^n$ a Boolean *affine space*. $\overline{xy}$ are the *free vectors*.

Let us define for any points $x, y, z, w \in \boldsymbol{B}_2^n, x \neq y, z \neq w$, the *parallelism*

(1.1) $$\{x, y\} \| \{z, w\} \Leftrightarrow \overline{xy} = \overline{zw} \Leftrightarrow x \oplus y \oplus z \oplus w = 0$$

This relation also shows that x, y, z, w are affinely dependent and that the vectors $\overline{xy}, \overline{xz}, \overline{xw}$ are linearly dependent. The set $\Pi = \{x, y, z, w\}$ where (1.1) is true is a plane (of dimension 2).

$\forall \{x, y\}$, the set $S \subset \{x, y\}, S \neq \varnothing$ is a *segment*. It's *length* is

(1.2) $$\| S \| = \begin{cases} 1, if \ \ cardinal \ S = 2 \\ 0, if \ \ cardinal \ S = 1 \end{cases}$$

$\forall x, y, z$ so that $x \neq y, x \neq z, \overline{yxz} \overset{def}{=} \begin{cases} \{x, \{y, z\}\}, if \ \ y \neq z \\ \{x, \{y\}\} \ \ \ , if \ \ y = z \end{cases}$ is an *angle* (or an *arc* as well). It's *measure* is:

(1.3) $$m(\overline{yxz}) = \begin{cases} 1, if \ \ y \neq z \\ 0, else \end{cases}$$

Fixing in $\Pi = \{x, y, z, w\}$ a point x means choosing an ***origin of the coordinates***; fixing an $y \in \Pi - \{x\}$ means choosing an *origin of the angles,* the line $\{x, y\}$. Fixing an $z \in \Pi - \{x, y\}$ means choosing an *orientation*.

In fig 1, (0,0) is the origin of the coordinates, {(0,0),(1,0)} is the origin of the angles and the orientation was precised.

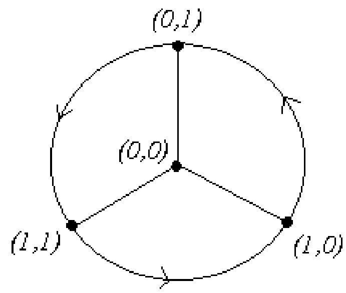

fig 1

Now it is possible to define the measure of the angles in the oriented plane by:

(1.4) $$m(\overline{(1,0)(0,0)(1,0)}) = 0$$
$$m(\overline{(1,0)(0,0)(0,1)}) = 1$$
$$m(\overline{(1,0)(0,0)(1,1)}) = 2$$

and it is seen that 0,1,2 are integers or modulo 3 classes, the same role having here the integer 3 like the number 2π in the real geometry.

The fundamental ***trigonometrical functions*** are

φ	s	c
0	0	1
1	1	0
2	1	1

table 1

that satisfy

(1.5) $$s(\varphi_1 + \varphi_2) = s(\varphi_1)\cdot c(\varphi_2) \oplus s(\varphi_2)\cdot c(\varphi_1) \oplus s(\varphi_1)\cdot s(\varphi_2)$$
$$s(\varphi_1 - \varphi_2) = s(\varphi_1)\cdot c(\varphi_2) \oplus s(\varphi_2)\cdot c(\varphi_1)$$
$$c(\varphi_1 + \varphi_2) = c(\varphi_1)\cdot c(\varphi_2) \oplus s(\varphi_1)\cdot s(\varphi_2)$$
$$c(\varphi_1 - \varphi_2) = c(\varphi_1)\cdot c(\varphi_2) \oplus s(\varphi_1)\cdot s(\varphi_2) \oplus c(\varphi_1)\cdot s(\varphi_2)$$

Here, '+' is the sum of the integers or the modulo 3 summation.

Fig 1 represents also a circle with the center in (0,0) (and the radius 1). It may be written in the form:

(1.6) $$C((0,0)) = \{x \mid x \in \boldsymbol{B}_2^2, \|\{x,(0,0)\}\| = 1\}$$
$$C((0,0)) = \{(x_1, x_2) \mid x_1 = c(\varphi), x_2 = s(\varphi), \varphi \in \boldsymbol{Z}\}$$

The ***sphere*** with the center in (0,0,0) (and radius 1) is

(1.7) $$C((0,0,0)) = \{x \mid x \in \boldsymbol{B}_2^3, \|\{x,(0,0,0)\}\| = 1\}$$
$$C((0,0,0)) = \{(x_1, x_2, x_3) \mid x_1 = s(\varphi), x_2 = c(\varphi)\cdot s(\theta), x_3 = c(\varphi)\cdot c(\theta), \varphi, \theta \in \boldsymbol{Z}\}$$

$\varphi = ct$ and $\theta = ct$ represent circles, $\varphi \neq 1$. $\varphi = 1$ represents the (unique) ***pole*** of the sphere. It is seen the position of the points (x_1, x_2, x_3) of the sphere when we associate them the values $x_1 \cdot 2^2 + x_2 \cdot 2^1 + x_3 \cdot 2^0$ (sum of integers).

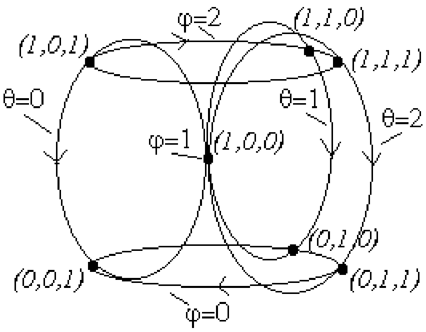

fig 2

2) The *complex numbers* are defined by:

(2.1) $$\forall z \in \boldsymbol{C}_2 = \boldsymbol{B}_2^2, z = (x, y) \overset{not}{=} x \oplus i \cdot y$$

and if $z \neq 0$, then there exists a *trigonometrical form*

(2.2) $$z = c(\varphi) \oplus i \cdot s(\varphi), \varphi \in \boldsymbol{Z}$$

(see (1.6)). The laws of addition and multiplication are:

(2.3) $$(x \oplus i \cdot y) \oplus (x' \oplus i \cdot y') = x \oplus x' \oplus i \cdot (y \oplus y')$$

(2.4) $$(x \oplus i \cdot y) \cdot (x' \oplus i \cdot y') = x \cdot x' \oplus y \cdot y' \oplus i \cdot (x \cdot y' \oplus y \cdot x' \oplus y \cdot y')$$

and from (2.4) we infer that

(2.5) $$(c(\varphi) \oplus i \cdot s(\varphi)) \cdot (c(\varphi') \oplus i \cdot s(\varphi')) = c(\varphi + \varphi') \oplus i \cdot s(\varphi + \varphi')$$

For a complex number $z \in \boldsymbol{C}_2$ with a non-null *modulus*:

(2.6) $$|z| = x \cup y$$

the *inverse element* is:

(2.7) $$z^{-1} = (x \oplus i \cdot y)^{-1} = x \oplus y \oplus i \cdot y = c(-\varphi) \oplus i \cdot s(-\varphi)$$

We remark that

(2.8) $$\forall z \neq 0, z^n = \begin{cases} 1, n = 3k \\ z, n = 3k + 1 \\ z^{-1}, n = 3k + 2 \end{cases}, n, k \in \boldsymbol{Z}$$

Bibliography

[1] R. Miron, D. Branzei, The Fundamentals of the arithmetics and Geometry (in Romanian), ed. Academiei RSR, Bucharest, 1983