On the Inertia of the Asynchronous Circuits

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Abstract

By making use of the notions and the notations from [12], we present the bounded delays, the absolute inertia and the relative inertia.

1 Bounded Delays

Theorem 1.1 *The next system*

$$\bigcap_{\xi\in[t-d_r,t-d_r+m_r]} u(\xi) \le x(t) \le \bigcup_{\xi\in[t-d_f,t-d_f+m_f]} u(\xi) \tag{1}$$

where $u,x \in S$ *and* $0 \le m_r \le d_r, 0 \le m_f \le d_f$ *defines a DC if and only if*

$$d_r \ge d_f - m_f, d_f \ge d_r - m_r \tag{2}$$

Proof *The proof consists in showing that (2) implies for any* u *the existence of a solution* x *of (1); any such* x *satisfies* $x \in Sol_{SC}(u)$*. If (2) is not fulfilled, it is proved that* u *exists so that (1) has no solutions.*

Definition 1.2 *The system (1), when (2) is true, is called the bounded delay condition (BDC).* u, x *are the input, respectively the state (or the output);* m_r, m_f *are the (rising, falling) memories (or thresholds for cancellation) and* d_r, d_f *, respectively* $d_f - m_f, d_r - m_r$ *are the (rising, falling) upper bounds, respectively the (rising, falling) lower bounds of the transmission delay for transitions. We say that the tuple* (u, m_r, d_r, m_f, d_f) *satisfies BDC. We shall also call bounded delay condition the function* $Sol_{BDC}^{m_r,d_r,m_f,d_f} : S \to P^*(S)$ *defined by*

$$Sol_{BDC}^{m_r,d_r,m_f,d_f}(u) = \{x \mid (u, m_r, d_r, m_f, d_f) \text{ satisfies } BDC\}$$

Definition 1.3 *The inequalities (2) are called the consistency condition (CC) of BDC.*

Theorem 1.4 *Let* $0 \le m_r \le d_r, 0 \le m_f \le d_f$ *and* $0 \le m'_r \le d'_r, 0 \le m'_f \le d'_f$ *so that CC is fulfilled for each of them.*

a) We note $d''_r = \min(d_r, d'_r), d''_f = \min(d_f, d'_f), m''_r = d''_r - \max(d_r - m_r, d'_r - m'_r),$ $m''_f = d''_f - \max(d_f - m_f, d'_f - m'_f)$. The next statements are equivalent:

a.i) $\forall u, Sol_{BDC}^{m_r,d_r,m_f,d_f}(u) \wedge Sol_{BDC}^{m'_r,d'_r,m'_f,d'_f}(u) \neq \emptyset$

a.ii) $d''_r \geq d''_f - m''_f, d''_f \geq d''_r - m''_r$

and if one of them is satisfied, then we have

$$Sol_{BDC}^{m_r,d_r,m_f,d_f} \wedge Sol_{BDC}^{m'_r,d'_r,m'_f,d'_f} = Sol_{BDC}^{m''_r,d''_r,m''_f,d''_f}$$

b) We use the notations $d''_r = \max(d_r, d'_r), d''_f = \max(d_f, d'_f), m''_r = d''_r - \min(d_r - m_r, d'_r - m'_r), m''_f = d''_f - \min(d_f - m_f, d'_f - m'_f)$. The inequalities $d''_r \geq d''_f - m''_f, d''_f \geq d''_r - m''_r$ are satisfied and

$$Sol_{BDC}^{m_r,d_r,m_f,d_f} \vee Sol_{BDC}^{m'_r,d'_r,m'_f,d'_f} \subset Sol_{BDC}^{m''_r,d''_r,m''_f,d''_f}$$

The previous inclusion becomes equality if and only if

$$\forall u, Sol_{BDC}^{m_r,d_r,m_f,d_f}(u) \wedge Sol_{BDC}^{m'_r,d'_r,m'_f,d'_f}(u) \neq \emptyset$$

c) The next statements are equivalent:

c.i) $Sol_{BDC}^{m_r,d_r,m_f,d_f}$ is deterministic

c.ii) The upper bounds and the lower bounds of the delays coincide:

$$d_r = d_f - m_f, d_f = d_r - m_r$$

c.iii) The memories are null

$$m_r = m_f = 0$$

c.iv) The bounded delay degenerates in a translation

$$\exists d \geq 0, Sol_{BDC}^{m_r,d_r,m_f,d_f} = I_d \tag{3}$$

d) The next statements are equivalent

d.i) $Sol_{BDC}^{m_r,d_r,m_f,d_f} \subset Sol_{BDC}^{m'_r,d'_r,m'_f,d'_f}$

d.ii) $d'_r - m'_r \leq d_r - m_r \leq d_f \leq d'_f, \; d'_f - m'_f \leq d_f - m_f \leq d_r \leq d'_r$

e) $Sol_{BDC}^{m_r,d_r,m_f,d_f}$ is time invariant

f) The next statements are equivalent

f.i) $Sol_{BDC}^{m_r,d_r,m_f,d_f}$ is symmetrical

f.ii) $d_r = d_f, m_r = m_f$

g) $Sol_{BDC}^{m_r+m'_r,d_r+d'_r,m_f+m'_f,d_f+d'_f}$ is a BDC and we have

$$Sol_{BDC}^{m'_r,d'_r,m'_f,d'_f} \circ Sol_{BDC}^{m_r,d_r,m_f,d_f} = Sol_{BDC}^{m_r+m'_r,d_r+d'_r,m_f+m'_f,d_f+d'_f}$$

2 Fixed and Inertial Delays

Definition 2.1 *Let* $u, x \in S$ *and* $d \ge 0$*. The equation (see 1.4 (3))*

$$x(t) = u(t - d)$$

is called the fixed delay condition (FDC). The delay defined by this equation is also called pure, ideal or non-inertial. A delay different from FDC is called inertial.

Corollary 2.2 *FDC is deterministic, time invariant, constant and symmetrical. The serial connection of the FDC's coincides with the composition of the translations:*

$$I_d \circ I_{d'} = I_{d'} \circ I_d = I_{d+d'}, d \ge 0, d' \ge 0$$

Remark 2.3 *At 2.1 inertia was defined to be the property of the DC's of being not ideal. In particular the non-deterministic DC's, for example the non-trivial BDC's (i.e. the BDC's with memory* $m_r + m_f > 0$*) are inertial.*

3 Absolute Inertia

Definition 3.1 *The property*

$$\overline{x(t-0)} \cdot x(t) \le \bigcap_{\xi \in [t, t+\delta_r]} x(\xi)$$

$$x(t-0) \cdot \overline{x(t)} \le \bigcap_{\xi \in [t, t+\delta_f]} \overline{x(\xi)}$$

true for $\delta_r \ge 0, \delta_f \ge 0$ *is called the absolute inertial condition (AIC), or the non-zenoness condition.* δ_r, δ_f *are called inertial parameters. If it is fulfilled, we say that the tuple* (δ_r, δ_f, x) *satisfies AIC. We also call AIC the set* $Sol_{AIC}^{\delta_r, \delta_f} \subset S$ *defined by*

$$Sol_{AIC}^{\delta_r, \delta_f} = \{x | (\delta_r, \delta_f, x) \text{ satisfies } AIC\}$$

Remark 3.2 *AIC means that if* x *switches from* 0 *to* 1*, then it remains* 1 *at least* $\delta_r \ge 0$ *time units + the dual property. To be remarked the trivial situation* $\delta_r = \delta_f = 0$*.*

Definition 3.3 *Let* i *a DC satisfying* $\forall u, i(u) \cap Sol_{AIC}^{\delta_r, \delta_f} \ne \emptyset$*. The DC* $i \cap Sol_{AIC}^{\delta_r, \delta_f}$ *is called absolute inertial delay condition (AIDC).* $Sol_{BDC}^{m_r, d_r, m_f, d_f} \cap Sol_{AIC}^{\delta_r, \delta_f}$ *is called bounded absolute inertial delay condition (BAIDC).*

Theorem 3.4 *The numbers* $0 \le m_r \le d_r, 0 \le m_f \le d_f$ *with CC true and* $\delta_r \ge 0, \delta_f \ge 0$ *are given. The next statements are equivalent:*

a) $\forall u, Sol_{BDC}^{m_r, d_r, m_f, d_f}(u) \cap Sol_{AIC}^{\delta_r, \delta_f} \ne \emptyset$

b) $\delta_r + \delta_f \le m_r + m_f$

Corollary 3.5 *$0 \le m_r \le d_r, 0 \le m_f \le d_f$, $0 \le m'_r \le d'_r, 0 \le m'_f \le d'_f$ and $\delta_r \ge 0, \delta_f \ge 0, \delta'_r \ge 0, \delta'_f \ge 0$ satisfy $d_r \ge d_f - m_f, d_f \ge d_r - m_r, d'_r \ge d'_f - m'_f, d'_f \ge d'_r - m'_r, \delta_r + \delta_f \le m_r + m_f, \delta'_r + \delta'_f \le m'_r + m'_f$. In such conditions $Sol_{BDC}^{m_r,d_r,m_f,d_f} \wedge Sol_{AIC}^{\delta_r,\delta_f}$, $Sol_{BDC}^{m'_r,d'_r,m'_f,d'_f} \wedge Sol_{AIC}^{\delta'_r,\delta'_f}$, $Sol_{BDC}^{m_r+m'_r,d_r+d'_r,m_f+m'_f,d_f+d'_f} \wedge Sol_{AIC}^{\delta'_r,\delta'_f}$ are BAIDC's and the next property of the serial connection holds:*

$$(Sol_{BDC}^{m'_r,d'_r,m'_f,d'_f} \wedge Sol_{AIC}^{\delta'_r,\delta'_f}) \circ (Sol_{BDC}^{m_r,d_r,m_f,d_f} \wedge Sol_{AIC}^{\delta_r,\delta_f}) \subset$$

$$\subset Sol_{BDC}^{m_r+m'_r,d_r+d'_r,m_f+m'_f,d_f+d'_f} \wedge Sol_{AIC}^{\delta'_r,\delta'_f}$$

4 Relative Inertia

Definition 4.1 *$0 \le \mu_r \le \delta_r, 0 \le \mu_f \le \delta_f$ and $u, x \in S$ are given. The property*

$$\overline{x(t-0)} \cdot x(t) \le \bigcap_{\xi \in [t-\delta_r, t-\delta_r+\mu_r]} u(\xi)$$

$$x(t-0) \cdot \overline{x(t)} \le \bigcap_{\xi \in [t-\delta_f, t-\delta_f+\mu_f]} \overline{u(\xi)}$$

is called the relative inertial condition (RIC). $\mu_r, \delta_r, \mu_f, \delta_f$ are called inertial parameters. If it is fulfilled, we say that the tuple $(u, \mu_r, \delta_r, \mu_f, \delta_f, x)$ satisfies RIC. We also call RIC the function $Sol_{RIC}^{\mu_r,\delta_r,\mu_f,\delta_f} : S \to P^*(S)$ defined by

$$Sol_{RIC}^{\mu_r,\delta_r,\mu_f,\delta_f}(u) = \{x | (u, \mu_r, \delta_r, \mu_f, \delta_f, x) \text{ satisfies RIC}\}$$

Theorem 4.2 *Let $0 \le \mu_r \le \delta_r, 0 \le \mu_f \le \delta_f, u \in S$ and $x \in Sol_{RIC}^{\mu_r,\delta_r,\mu_f,\delta_f}(u)$ arbitrary. If $\delta_r \ge \delta_f - \mu_f, \delta_f \ge \delta_r - \mu_r$ then $x \in Sol_{AIC}^{\delta_f-\delta_r+\mu_r,\delta_r-\delta_f+\mu_f}$.*

Remark 4.3 *RIC states that the inertial delays 'model the fact that the practical circuits will not respond (at the output) to two transitions (at the input) which are very close together' [1], [2]. Theorem 4.2 connecting AIC and RIC makes use of the condition $\delta_r \ge \delta_f - \mu_f, \delta_f \ge \delta_r - \mu_r$ that is very similar to CC, but with a different meaning.*

Definition 4.4 *Let i a DC with $\forall u, i(u) \wedge Sol_{RIC}^{\mu_r,\delta_r,\mu_f,\delta_f}(u) \ne \emptyset$. Then the DC $i \wedge Sol_{RIC}^{\mu_r,\delta_r,\mu_f,\delta_f}$ (see Theorem 4.4 c) in [12]) is called relative inertial delay condition (RIDC). In particular $Sol_{BDC}^{m_r,d_r,m_f,d_f} \wedge Sol_{RIC}^{\mu_r,\delta_r,\mu_f,\delta_f}$ is called bounded relative inertial delay condition (BRIDC).*

Theorem 4.5 *Let the numbers $0 \le m_r \le d_r, 0 \le m_f \le d_f$. The next conditions are equivalent*

a) $\forall u, Sol_{BDC}^{m_r,d_r,m_f,d_f}(u) \cap Sol_{RIC}^{\mu_r,\delta_r,\mu_f,\delta_f}(u) \neq \emptyset$;

b) One of the next conditions is true

b.i) $d_f - m_f \leq \delta_r \leq d_r \leq \delta_r - \mu_r + m_r, d_r - m_r \leq \delta_f \leq d_f \leq \delta_f - \mu_f + m_f$

b.ii) $d_r - m_r + \mu_r \leq \delta_r \leq d_f - m_f \leq d_r, d_f - m_f + \mu_f \leq \delta_f \leq d_r - m_r \leq d_f$

b.iii) $d_f - m_f \leq \delta_r \leq d_r - m_r + \mu_r \leq d_r, d_r - m_r \leq \delta_f \leq d_f - m_f + \mu_f \leq d_f$

b.iv) $\delta_r \leq d_f - m_f \leq \delta_r + m_r - \mu_r \leq d_r, \delta_f \leq d_r - m_r \leq \delta_f + m_f - \mu_f \leq d_f$

Remark 4.6 *The equivalent conditions from Theorem 4.5 are of consistency of BRIDC, they are stronger than CC (of BDC) and weaker than (see the hypothesis $\delta_r \geq \delta_f - \mu_f, \delta_f \geq \delta_r - \mu_r$ from Theorem 4.2)*

$$d_f - m_f \leq \delta_f - \mu_f \leq \delta_r \leq d_r$$
$$d_r - m_r \leq \delta_r - \mu_r \leq \delta_f \leq d_f$$

Theorem 4.7 *Let $0 \leq m_r \leq d_r, 0 \leq m_f \leq d_f$ so that CC is fulfilled and $u \in S$ arbitrary. The next statements are equivalent:*

a) $x \in Sol_{BDC}^{m_r,d_r,m_f,d_f}(u) \cap Sol_{RIC}^{m_r,d_r,m_f,d_f}(u)$

b)

$$\overline{x(t-0)} \cdot x(t) = \overline{x(t-0)} \cdot \bigcap_{\xi \in [t-d_r, t-d_r+m_r]} u(\xi)$$
$$x(t-0) \cdot \overline{x(t)} = x(t-0) \cdot \bigcap_{\xi \in [t-d_f, t-d_f+m_f]} \overline{u(\xi)}$$

Theorem 4.8 *Any of the previous equivalent conditions defines a deterministic, time invariant, constant DC.*

Remark 4.9 *The deterministic situation 4.7 of BRIDC has as special case I_d, happening when $m_r = m_f = 0, d_r = d_f = d$. On the other hand the serial connection of the BRIDC's is not a BRIDC. We also mention the possibility of replacing the functions $\bigcap_{\xi \in [t-d_r, t-d_r+m_r]} u(\xi)$, $\bigcup_{\xi \in [t-d_f, t-d_f+m_f]} u(\xi)$ with $\bigcap_{\xi \in [t-d_r, t)} u(\xi)$, $\bigcup_{\xi \in [t-d_f, t)} u(\xi)$ in BDC, the functions $\bigcap_{\xi \in [t, t+\delta_r]} x(\xi)$, $\bigcap_{\xi \in [t, t+\delta_f]} \overline{x(\xi)}$ with $\bigcap_{\xi \in [t, t+\delta_r)} x(\xi)$, $\bigcap_{\xi \in [t, t+\delta_f)} \overline{x(\xi)}$ in AIC, the functions $\bigcap_{\xi \in [t-\delta_r, t-\delta_r+\mu_r]} u(\xi)$ and $\bigcap_{\xi \in [t-\delta_f, t-\delta_f+\mu_f]} \overline{u(\xi)}$ with $\bigcap_{\xi \in [t-\delta_r, t)} u(\xi)$, $\bigcap_{\xi \in [t-\delta_f, t)} \overline{u(\xi)}$ in RIC etc. and some variants of the previous definitions result. The last six functions are not signals.*

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